Title: Discretization of Laplacian operator

Abstract: Laplacian operator is of very important in basic sciences and engineerings. From this fundamental operator it is possible to define the classical partial differential equations like the Laplace equation $\Delta f = 0$, Poisson equation $\Delta f = g$, Wave equation $\Delta f = \frac{\partial^2 f}{\partial t^2}$ and Heat equation $\Delta f = \frac{\partial f}{\partial t}$. In physics, this operator permits to model diverse situations relatives to fluid mechanics, electromagnetism, quantum mechanics, among other.

From the method of finite differences it is possible to make an approximation of the Laplacian and bring partial differential equations with very difficult analytical solutions to determine the solution of a system of linear equations. A classic way of doing this is to discretize the domain through rectangular meshes in which all the rectangles have the same area A, and, in each node, calculate the difference of the value of the function and the values in the neighboring nodes. In two dimensions it is written as $(\Delta f)_i = \frac{1}{A}\sum_{j\sim i}(f_j - f_i)$ where $j \sim i$ indicates that the vertex j is neighbor to the vertex i, that is, the vertices to left, right, up and down.

Another method that allows this discretization is by means of the operator Laplaciano Cotangent, which acts on functions defined in the vertices of triangular meshes in three-dimensional space, and which is given by $(\Delta f)_i = \frac{1}{2A_f}\sum_{j\sim i}\left(\cot\alpha_{ij} + \cot\beta_{ij}\right)(f_j - f_i)$ where A_f is one third of the sum of the areas of the triangles of the mesh for which i is one of its vertices, and α_{ij}, β_{ij} are the angles opposite the edge i-j, of the triangles that have the edge i-j as common edge.

This discretization allows to find the geodetic distance in a manifold M. For this the following three steps are followed: i) Obtain a solution to the heat equation, ii) normalize the gradient of the solution to the heat equation, and, finally iii) solve a Poisson equation that involves the geodetic distance and the divergence of the vector field previously found. This method, called "Heat Method", was presented by Keenan Crane in 2012 and is currently applied in the solution of different problems.