Hybrid approaches to the Repositioning Problem in Bicycle-Sharing Systems

Juan David Palacio Domínguez
PhD Student in Mathematical Engineering

Juan Carlos Rivera Agudelo
Thesis Advisor

Doctoral Seminar in Mathematical Engineering

UNIVERSIDAD EAFIT

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Outline

1. The Repositioning Problem (RP) – Description
2. Solution strategies
3. Hybrid algorithms in combinatorial optimization
4. Preliminary results
5. Current and future work


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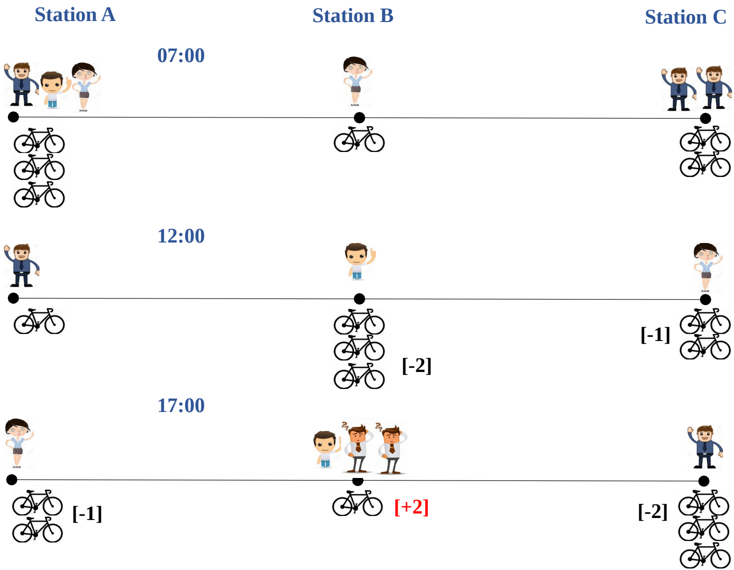
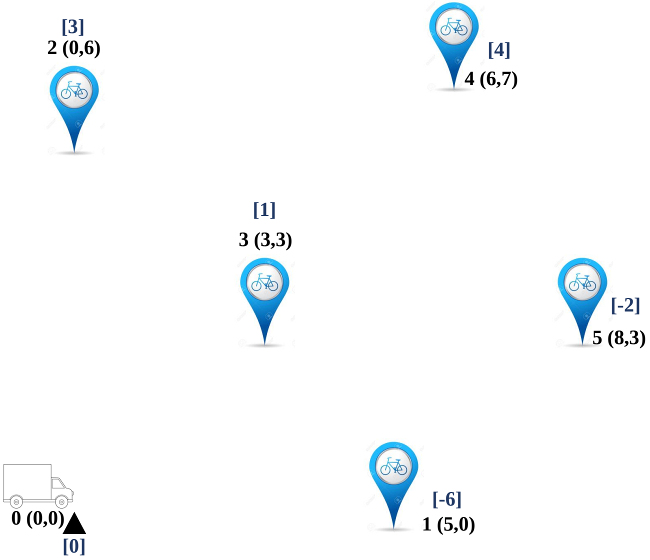
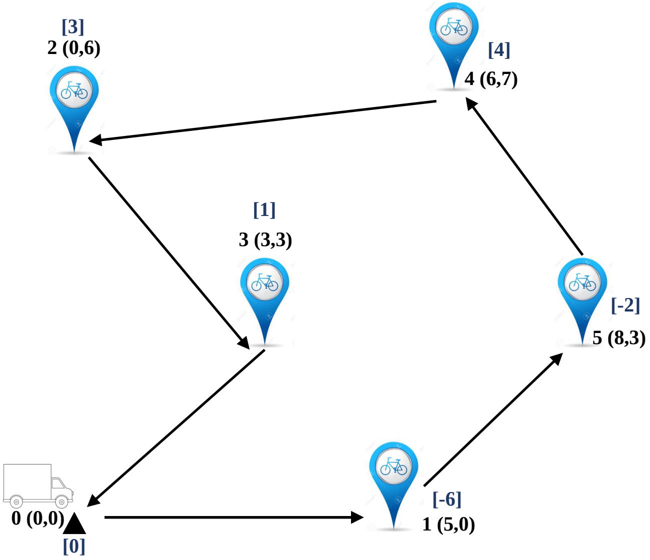
Balancing a BSS



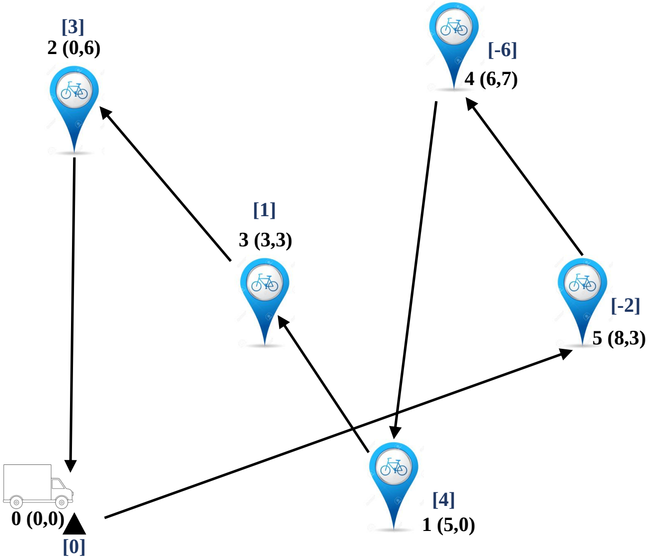
Pick up and Delivery TSP



Pick up and Delivery TSP



Pick up and Delivery TSP



Solution strategies – Single vehicle case

- Mixed Integer Programming Models (MILPs)
  - Traveling Salesman Problem (TSP)
  - Pick up and Delivery TSP (PDTSP)
  - PDTSP with Split Demand (PDTSPSD)
- Heuristic algorithms
  - Nearest Neighbor (TSP)
  - Extensions of Nearest Neighbor for PDTSP and PDTSPSD
- Metaheuristic algorithms
  - Greedy Randomized Adaptive Search Procedure (GRASP)
  - Path Relinking
  - Variable Neighborhood Descent (VND)
- Hybrid approaches (matheuristics)
  - MILP based local search operators
  - MILP based post-optimization procedures
  - PDTPSP decomposition

A GRASP algorithm

Algorithm 1 GRASP

$f^* \leftarrow \infty$
for $i = 1$ to *GRASPIterations* **do**
  $S \leftarrow$GreedyRandomAlgorithm()
  $S \leftarrow$LocalSearch(S)
  if $f(S) < f^*$ **then**
    $S^* \leftarrow S$
    $f^* \leftarrow f(S)$
  end if
end for
return S^*

Our GRASP algorithm

Algorithm 2 GRASP + VND

$f^* \leftarrow \infty$
for $i = 1$ to *GRASPIterations* **do**
  $S \leftarrow$GreedyRandomAlgorithm()
  $S \leftarrow$VND(S)
  if $f(S) < f^*$ **then**
    $S^* \leftarrow S$
    $f^* \leftarrow f(S)$
  end if
end for
return S^*

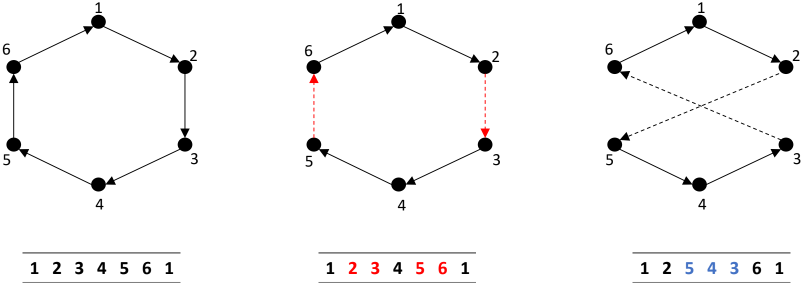
VND Structure

Six neighborhoods within a VND method

- Destroy and Repair
- Lin & Kernighan algorithm
- Or-opt(λ), $\lambda = 2, 3$
- Forward insertion
- Backward insertion

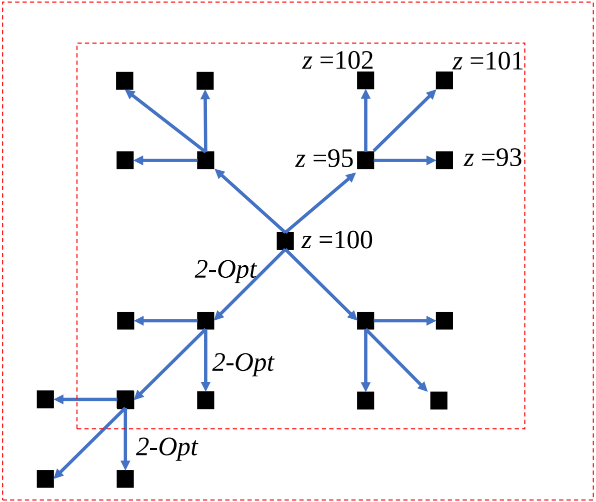
Lin & Kernighan algorithm

The Lin & Kernighan algorithm is based on 2-opt operator



Lin & Kernighan algorithm

Being z the total distance for a PDTSP solution:



Or-opt(λ), $\lambda = 2, 3$

Move forward or backward λ nodes (some movements are infeasibles):

Path	**1**	**2**	**3**	**4**	**5**	**6**	**1**
Demand	0	-5	2	-4	3	4	0

Or -opt (2)

Path	**1**	**2**	**3**	**4**	**5**	**6**	**1**
Demand	0	-5	2	-4	3	4	0
Path	**1**	**4**	**5**	**2**	**3**	**6**	**1**
Demand	0	-4	3	-5	2	4	0

Or -opt (3)

Path	**1**	**2**	**3**	**4**	**5**	**6**	**1**
Demand	0	-5	2	-4	3	4	0
Path	**1**	**4**	**5**	**6**	**2**	**3**	**1**
Demand	0	-4	3	4	-5	2	0

Hybrid algorithms in combinatorial optimization

Exact approaches

Strategies able to provide the optimal solution for an optimization problem. E.g.: Linear Programming, Mixed (and) Integer Programming, Dynamic Programming.

(Meta)Heuristic algorithms

Strategies able to provide good (*near to optimal*) solutions for an optimization problem in a decent computational time. E.g.: Greddy Randomized Adaptive Search Procedure (GRASP), Local Search (LS).

Hybrid algorithms (matheuristics)

Exact approaches + (Meta)Heuristic algorithms

A GRASP algorithm

Algorithm 3 GRASP

$f^* \leftarrow \infty$;
for $i = 1$ to *GRASPIterations* **do**
    $S \leftarrow$GreedyRandomAlgorithm();
    $S \leftarrow$LocalSearch(S);
    if $f(S) < f^*$ **then**
        $S^* \leftarrow S$;
        $f^* \leftarrow f(S)$;
    end if
end for
return S^*

Algorithm 4 A first GRASP based matheuristic

```
f* ← ∞;
for i = 1 to GRASPIterations do
    S ←GreedyRandomAlgorithm();
    S ←SolveMILP(S);  //A MILP as intesification procedure
    if f(S) < f* then
        S* ← S;
        f* ← f(S);
    end if
end for
return  S*
```

A GRASP based matheuristic – An example

Algorithm 5 A second GRASP based matheuristic

$f^* \leftarrow \infty$;
for $i = 1$ to *GRASPIterations* **do**
  $S \leftarrow$GreedyRandomAlgorithm();
  $S \leftarrow$LocalSearch(S);
  if $f(S) < f^*$ **then**
    $S^* \leftarrow S$;
    $f^* \leftarrow f(S)$;
  end if
end for
$S^* \leftarrow$SolveMILP(S^*) // A MILP as post-optimization procedure
return S^*

Selective TSP-based matheuristic

Algorithm 6 Selective TSP-based matheuristic

$\mathcal{A} \leftarrow \emptyset$
while $|\mathcal{A}| < |\mathcal{N}|$ **do**
    $\mathcal{A} \leftarrow$ SolveSelectiveMILP($\mathcal{A}$)
end while
$S^* \leftarrow$BuildPath($\mathcal{A}$)
return S^*

MILP for the Selective TSP matheuristic (SolveSelectiveMILP)

- Sets
  - $\mathcal{N}$: Set of stations (including the depot)
  - $\mathcal{N}_s$: Set of stations (excluding the depot)
  - $\mathcal{A}$: Set of fixed arcs from previous solutions
- Parameters
  - c_{ij} : Cost of traveling from station i to station j
  - q_i : Demand or slack of bicycles in station i
  - Q : Vehicle capacity
- Decision Variables
  - $y_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is used in the solution} \\ 0 & \text{otherwise} \end{cases}$
  - x_{ij} : Vehicle load when traveling from i to j
  - z_{ij} : Position of arc (i,j) in the solution

MILP for the Selective TSP matheuristic (`SolveSelectiveMILP`)

$$\min f = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} c_{ij} \cdot y_{ij} \tag{1}$$

subject to,

$$\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} y_{ij} \geq |\mathcal{A}| + 1 \tag{2}$$

$$\sum_{k \in \mathcal{N}} z_{ik} - \sum_{k \in \mathcal{N}} z_{jk} \geq 1 \qquad \forall\ (i,j) \in \mathcal{A} \tag{3}$$

$$\sum_{j \in \mathcal{N},\ i \neq j} y_{ij} \leq 1 \qquad \forall\ i \in \mathcal{N} \tag{4}$$

$$\sum_{i \in \mathcal{N}} y_{ij} = \sum_{i \in \mathcal{N}} y_{ji} \qquad \forall\ j \in \mathcal{N} \tag{5}$$

MILP for the Selective TSP matheuristic (`SolveSelectiveMILP`)

$$x_{ij} \leq Q \cdot y_{ij} \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \tag{6}$$

$$\sum_{j \in \mathcal{N}} x_{ji} - \sum_{j \in \mathcal{N}} x_{ij} = q_i \cdot \sum_{j \in \mathcal{N}} y_{ij} \qquad \forall\ i \in \mathcal{N} \tag{7}$$

$$\sum_{j \in \mathcal{N}} z_{ji} - \sum_{j \in \mathcal{N}} z_{ij} = \sum_{j \in \mathcal{N}} y_{ij} \qquad \forall\ i \in \mathcal{N}_s \tag{8}$$

$$z_{ij} \leq |\mathcal{N}| \cdot y_{ij} \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \tag{9}$$

$$y_{ij} \in \{0, 1\} \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \tag{10}$$

$$z_{ij} \in \mathcal{Z}^+ \cup \{0\} \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \tag{11}$$

$$x_{ij} \geq 0 \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \tag{12}$$

Selective TSP + VND - based matheuristic

Algorithm 7 Selective TSP + VND – based matheuristic

$\mathcal{A} \leftarrow \emptyset$
while $|\mathcal{A}| < |\mathcal{N}|$ **do**
    $\mathcal{A} \leftarrow$ SolveSelectiveMIP($\mathcal{A}$)
end while
$S^* \leftarrow$BuildPath($\mathcal{A}$)
$S^* \leftarrow$VND(S^*)
return S^*

GRASP + MILP post-optimization procedure

Algorithm 8 GRASP + MILP post-optimization procedure

$f^* \leftarrow \infty$, $\xi \leftarrow \emptyset$
for $i = 1$ to *GRASPIterations* **do**
    $S \leftarrow$GreedyRandomAlgorithm()
    $S \leftarrow$VND(S);
    if $f(S) < f^*$ **then**
        $S^* \leftarrow S$;
        $f^* \leftarrow f(S)$;
    end if
    if isElite(S)=**true then**
        $\xi \leftarrow \xi \cup S$;
    end if
end for
$S^* \leftarrow$Improve(ξ);
return S^*;

Improve elite solution algorithm

Algorithm 9 Improve elite solutions algorithm

1: **for** $i = 1$ to $|\xi|$ **do**
2: $\mathcal{A} \leftarrow$Destroy($\xi_{[i]}$)
3: $S \leftarrow$RepairMILP($\mathcal{A}$)
4: **if** $f(S) < f^*$ **then**
5: $S^* \leftarrow S$;
6: $f^* \leftarrow f(S)$;
7: **end if**
8: **end for**
9: **return** S^*;

MILP for the repairing function (`RepairMILP`)

$$\min f = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} c_{ij} \cdot y_{ij} \tag{13}$$

subject to,

$$\sum_{k \in \mathcal{N}} z_{ik} - \sum_{k \in \mathcal{N}} z_{jk} \geq 1 \qquad \forall\ (i,j) \in \mathcal{A} \tag{14}$$

$$\sum_{j \in \mathcal{N},\ i \neq j} y_{ij} = 1 \qquad \forall\ i \in \mathcal{N} \tag{15}$$

$$\sum_{i \in \mathcal{N}} y_{ij} = \sum_{i \in \mathcal{N}} y_{ji} \qquad \forall\ j \in \mathcal{N} \tag{16}$$

$$x_{ij} \leq Q \cdot y_{ij} \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \tag{17}$$

MILP for the repairing function (`RepairMILP`)

$$\sum_{j \in \mathcal{N}} x_{ji} - \sum_{j \in \mathcal{N}} x_{ij} = q_i \qquad \forall\ i \in \mathcal{N} \tag{18}$$

$$\sum_{j \in \mathcal{N}} z_{ji} - \sum_{j \in \mathcal{N}} z_{ij} = 1 \qquad \forall\ i \in \mathcal{N}_s \tag{19}$$

$$z_{ij} \leq |\mathcal{N}| \cdot y_{ij} \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \tag{20}$$

$$y_{ij} \in \{0, 1\} \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \tag{21}$$

$$z_{ij} \in \mathcal{Z}^+ \cup \{0\} \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \tag{22}$$

$$x_{ij} \geq 0 \qquad \forall\ i \in \mathcal{N}, j \in \mathcal{N} \tag{23}$$

- Dataset
  - Instances adapted from TSPLib Library (`elib.zib.de/pub/mp-testdata/tsp/tsplib/tsp/index.html`)
  - Instances with 9, 14, 16, 22, 29, 42 nodes were tested
- Software
  - All the algorithms were implemented on C++
  - Mathematical models were solved using Gurobi Optimizer 7.1
- Computer features
  - Intel core i7, 64Gb RAM.
  - OS: Linux - Debian 8 (x86-64)

Preliminary results – GRASP + VND

Table: GRASP + VND results with GRASPIterations=200 and 10 runs

		GRASP + VND				
Number stations	Optimal solution	Best solution	Best GAP	Average solution	Average GAP	Average time (s)
9	38.15	38.15	0.00%	38.15	0.00%	0.03
14	36.01	36.01	0.00%	36.01	0.00%	0.12
16	84.84	84.84	0.00%	84.84	0.00%	0.12
22	95.84	95.84	0.00%	95.84	0.00%	0.23
29	13529.2	13529.2	0.00%	13563.57	0.25%	0.46
42	1446.33	1446.33	0.00%	1451.48	0.36%	1.24

Preliminary results – Selective TSP matheuristic

Table: Selective TSP + VND matheuristic results

		Selective TSP + VND matheuristic					
Number stations	Optimal solution	Iterative phase (Selective TSP)			Selective TSP + VND		
		Solution	GAP	Time (s)	Solution	GAP	Time (s)
9	38.15	40.65	6.55%	1.10	38.15	0.00%	1.11
14	36.01	36.74	2.03%	10.69	36.01	0.00%	10.70
16	84.84	85.50	0.78%	31.84	84.84	0.00%	31.87
22	95.84	116.69	21.76%	260.33	106.08	10.68%	260.35
29	13529.2	17210.6	27.21%	206.71	16108.1	19.06%	207.24
42	1446.33	1855.9	29.32%	442.92	1579.86	9.23%	442.97

current and future work

- Improve the performance of the matheuristic algorithms by designing new decomposition strategies.
- Solve larger instances of the PDTSP via GRASP + VND and compare our results with reported benchmarks.
- Extend our solutions algorithms to the multiple vehicle clase.

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Juan David Palacio Domínguez

jpalac26@eafit.edu.co

Juan Carlos Rivera Agudelo

jrivera6@eafit.edu.co