

Inspire Create Transform

Wavefield separation cross-correlation imaging condition based on continuous wavelet transform

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Outline

Introduction

Laguerre-Gauss transform in post-processing imaging

Wavefield decomposition

Continuous wavelet transform

Wavefield separation based on CWT

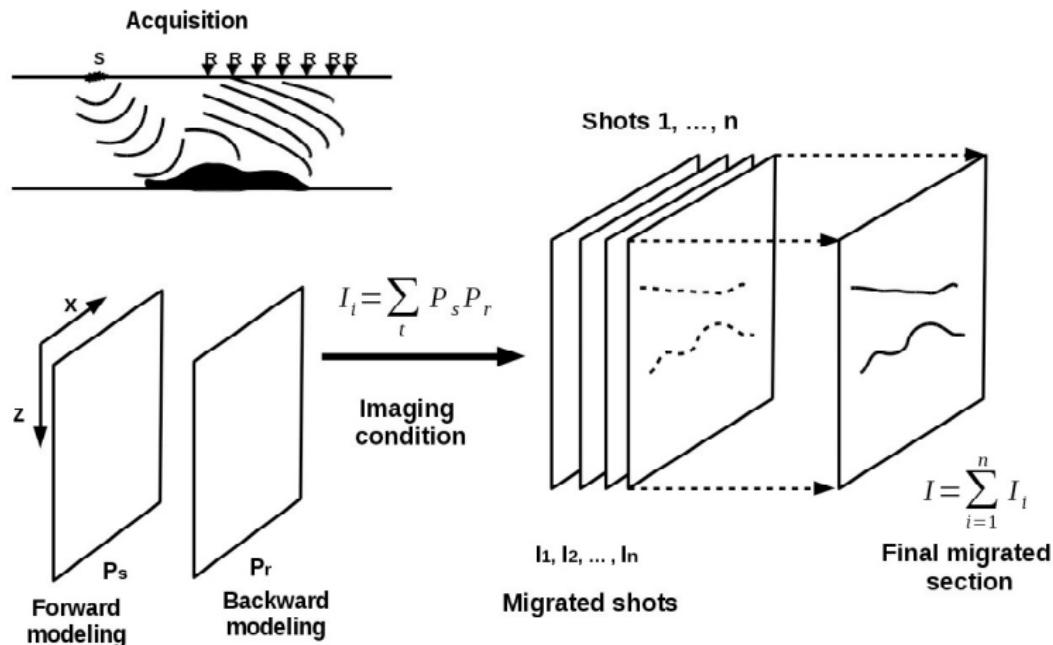
Outline

Preliminary results

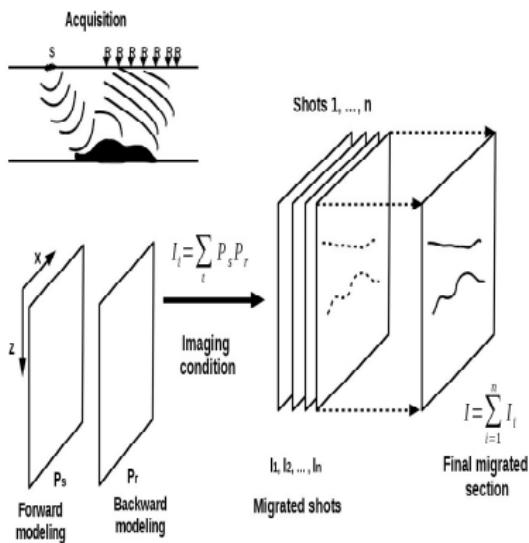
Future work

References

Reverse time migration (RTM)



Reverse time migration (RTM)

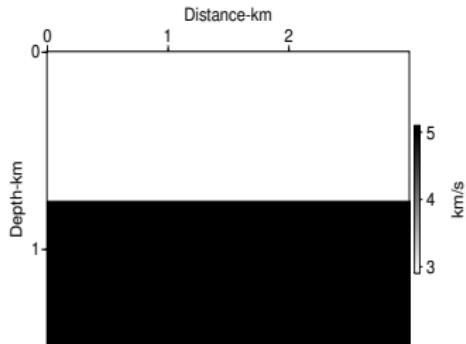


Acoustic wave equation

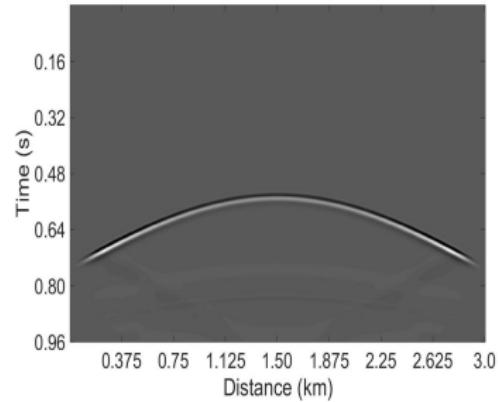
$$\frac{1}{c(x)^2} \frac{\partial^2 u(x, t)}{\partial t^2} - \nabla^2 u(x, t) = s(x, t)$$

1. Forward propagation of the source wavefield.
2. Backward propagation of the receivers wavefield.
3. Imaging condition.

Reverse time migration (RTM)

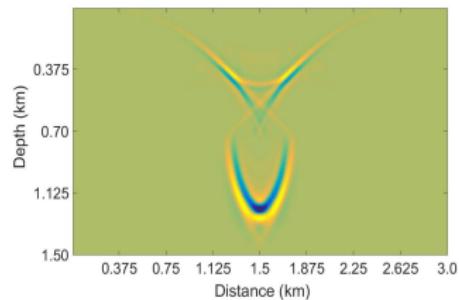
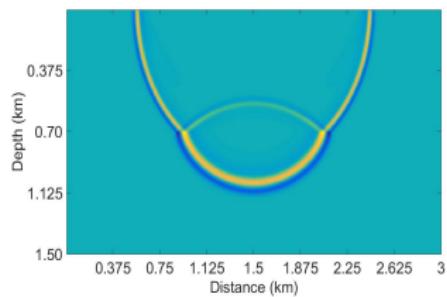
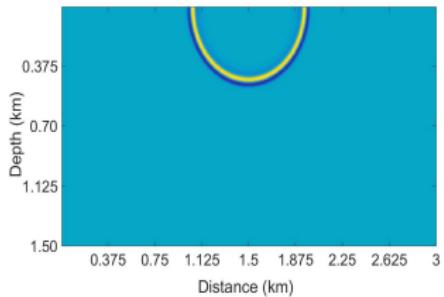


Velocity model

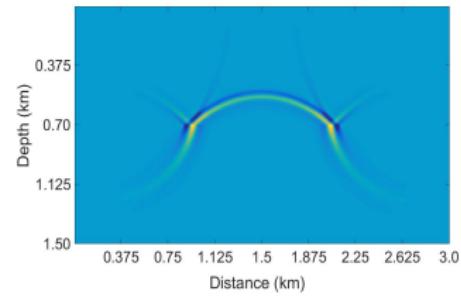


Data recorded

Source wavefield video Receiver wavefield video



$t = 0.20$ s



$t = 0.36$ s

Zero-lag cross-correlation imaging condition (ZL-CC-IC)

$$I_{cc}(x, z) = \sum_{j=1}^{s_{max}} \sum_{i=1}^{t_{max}} S(x, z; t_i; s_j) R(x, z; t_i; s_j) \quad (1)$$

S : Source wavefield

R : Receiver wavefield

z : Depth

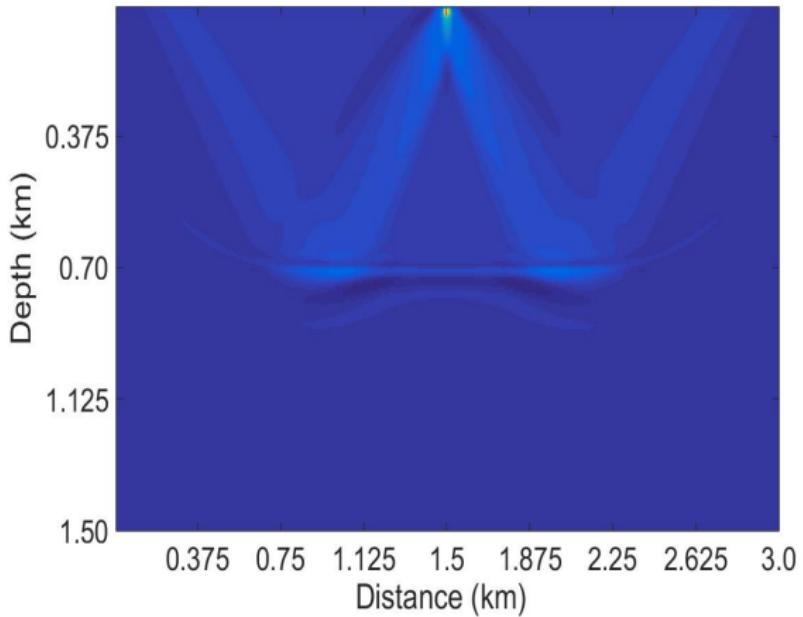
x : Distance

t : Time

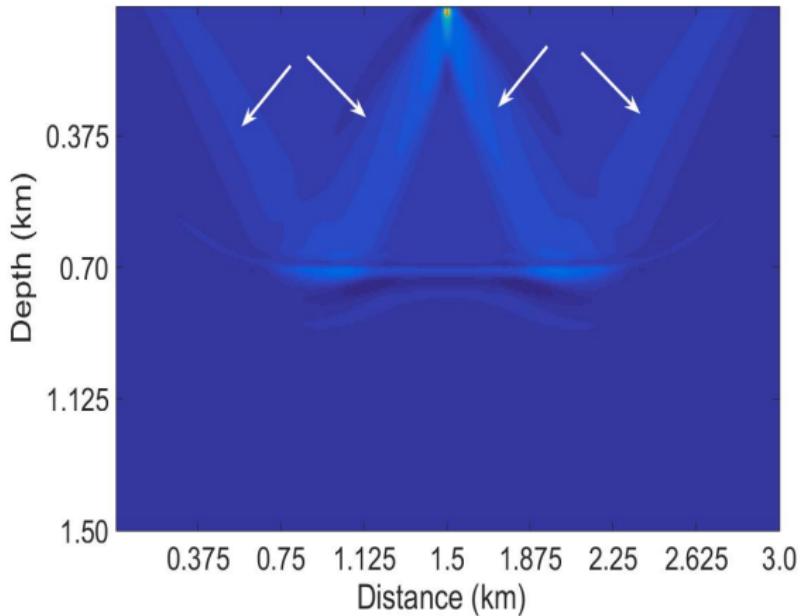
t_{max} : Maximum time

s_{max} : Maximum number of sources

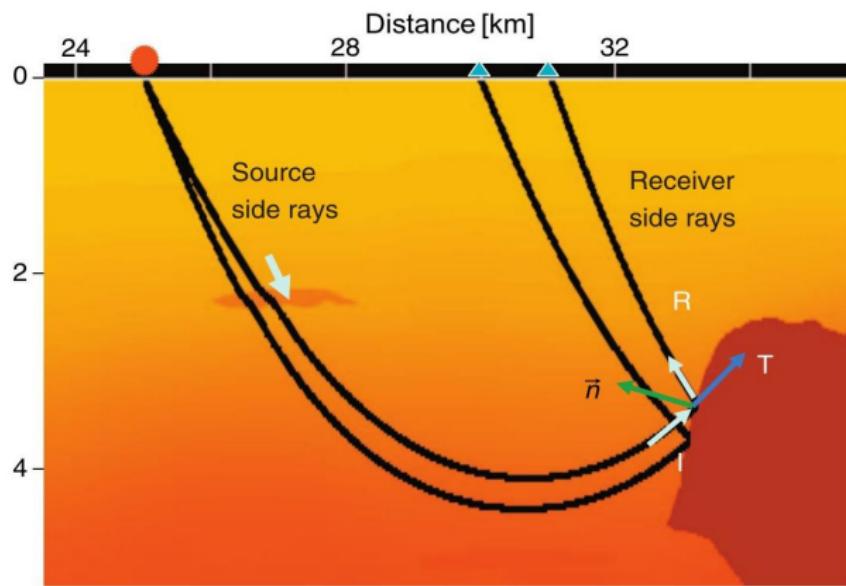
Cross-correlation image



Cross-correlation image



Some wave paths of the wavefield [22]



Methods to eliminate the artifacts

- ▶ Wavefield propagation approaches ([25, 3, 12]).
- ▶ Imaging condition approaches ([38, 20, 17, 22, 43, 31, 35]).
- ▶ Post-imaging condition approaches ([45, 16]).

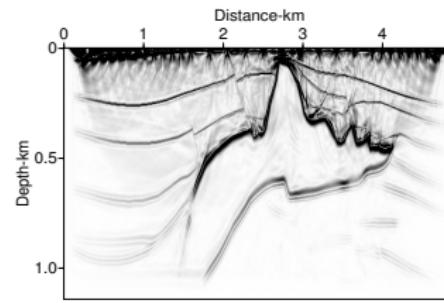
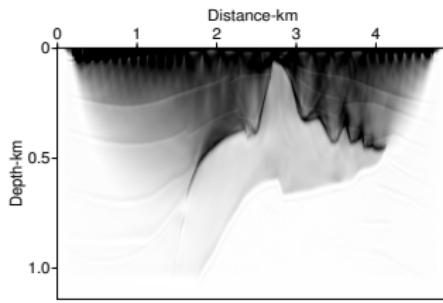
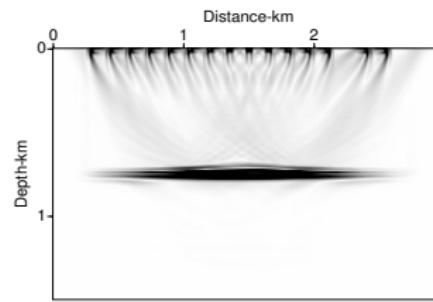
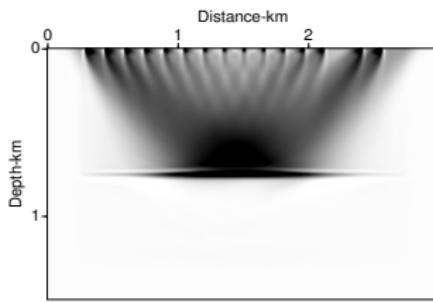
Laguerre-Gauss transform

The Laguerre-Gauss transform of $I(x, y)$ is given by ([41, 15])

$$\tilde{I}(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} LG(f_x, f_y) I(f_x, f_y) e^{2\pi i(f_x x + f_y y)} df_x df_y$$

Where

$$LG(x, y) = (i\pi^2 \omega^4)(x + iy) e^{-\pi^2 \omega^2(x^2 + y^2)}$$



Cross-correlation image

Laguerre-Gauss image

Wavefield decomposition

Taking into account (1)

$$I_{cc}(x, z) = \sum_{j=1}^{s_{max}} \sum_{i=1}^{t_{max}} S(x, z; t_i; s_j) R(x, z; t_i; s_j)$$

$S(x, z; t_i; s_j)$ and $R(x, z; t_i; s_j)$ can be partitioned mathematically as

$$S(x, z; t_i; s_j) = S_d(x, z; t_i; s_j) + S_u(x, z; t_i; s_j)$$

$$R(x, z; t_i; s_j) = R_d(x, z; t_i; s_j) + R_u(x, z; t_i; s_j)$$

Wavefield decomposition

Then, (1) can be expressed as follows

$$\begin{aligned} I_{cc}(x, z) = & \sum_{j=1}^{s_{max}} \sum_{i=1}^{t_{max}} (S_d(x, z; t_i; s_j) R_u(x, z; t_i; s_j) \\ & + S_u(x, z; t_i; s_j) R_d(x, z; t_i; s_j) \\ & + S_d(x, z; t_i; s_j) R_d(x, z; t_i; s_j) \\ & + S_u(x, z; t_i; s_j) R_u(x, z; t_i; s_j)) \end{aligned}$$

Then

$$I_{cc}(x, z) = I_{cc}^{du}(x, z) + I_{cc}^{ud}(x, z) + I_{cc}^{dd}(x, z) + I_{cc}^{uu}(x, z) \quad (2)$$

Wavefield decomposition

From (2)

$$I_{cc}(x, z) = I_{cc}^{du}(x, z) = \sum_{j=1}^{s_{\max}} \sum_{i=1}^{t_{\max}} S_d(x, z; t_i; s_j) R_u(x, z; t_i; s_j) \quad (3)$$

$S_d(x, z; t_i; s_j)$: Downgoing source wavefield.

$R_u(x, z; t_i; s_j)$: Upgoing receiver wavefield.

Eq. (3) is exactly what one will get in a one way wave equation migration

Continuous wavelet transform

A wavelet is a function $\psi \in L^2(\mathbb{R})$ with finite energy ([27]), that is,

$$C_\psi = \int_0^\infty \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty$$

$\hat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$ given by

$$\hat{\psi}(\omega) = \int_{-\infty}^{\infty} \psi(t) e^{-i2\pi\omega t} dt$$

C_ψ is called the admissibility condition.

Continuous wavelet transform

It is normalized $\|\psi\| = 1$ and satisfies the condition that is rapidly decreasing

$$\int_{-\infty}^{\infty} (1 + |t|) |\psi(t)| dt < \infty$$

with zero average and centered in the neighborhood of $t = 0$

$$\int_{-\infty}^{\infty} \psi(t) dt = 0$$

Continuous wavelet transform

Family of wavelets

$$\psi_{s,u}(t) = \frac{1}{\sqrt{s}} \psi \left(\frac{t-u}{s} \right), \quad s, u \in \mathbb{R}, \quad a \neq 0$$

s : Scaling parameter

u : Translation parameter

ψ : Mother wavelet

If $\psi \in L^2(\mathbb{R})$, then $\psi_{s,u}(t) \in L^2(\mathbb{R})$ for all s, u and $\|\psi_{s,u}\| = 1$.

Continuous wavelet transform

The integral transformation W_f defined on $L^2(\mathbb{R})$ by

$$W_f(u, s) = \langle f(t), \psi_{s,u}(t) \rangle = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t) \psi^* \left(\frac{t-u}{s} \right) dt$$

is called a continuous wavelet transform of $f(t)$.

Gaussian wavelet

$$\psi_n(t) = c_n \frac{d^n}{dt^n} \left(e^{-\frac{t^2}{2}} \right), \quad \hat{\psi}_n(\omega) = c_n (i\omega)^n e^{-\frac{\omega^2}{4}}$$

Continuous wavelet transform

The continuous wavelet transform can be expressed as a convolution product

$$W_f(u, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t) \psi^* \left(\frac{t-u}{s} \right) dt = f(t) \star \bar{\psi}_s(u)$$

with

$$\bar{\psi}_s(t) = \frac{1}{\sqrt{s}} \psi^* \left(\frac{-t}{s} \right)$$

and the Fourier transform of $\bar{\psi}_s(t)$ is

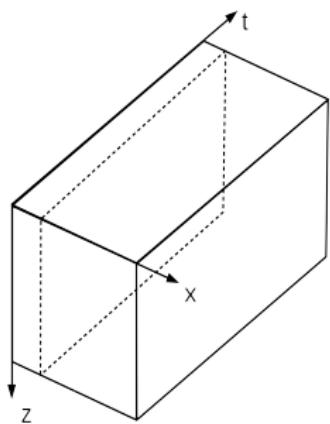
$$\hat{\bar{\psi}}_s(\omega) = \sqrt{s} \hat{\psi}^*(s\omega)$$

Source wavefield analysis

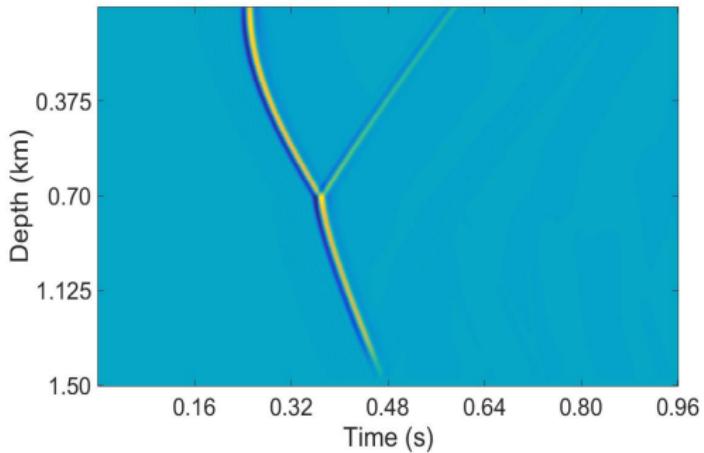
Algorithm for time-scale analysis of source wavefield

- ▶ From the source wavefield $S(x, z, t)$, select for each x the wavefield $S_x(z, t)$.

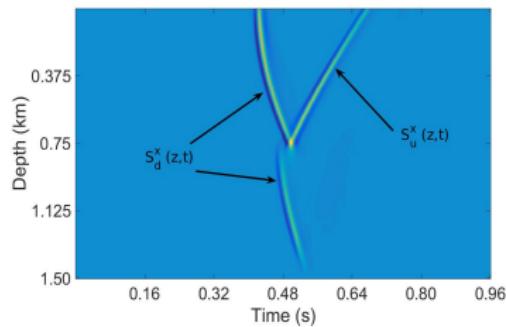
Source wavefield analysis



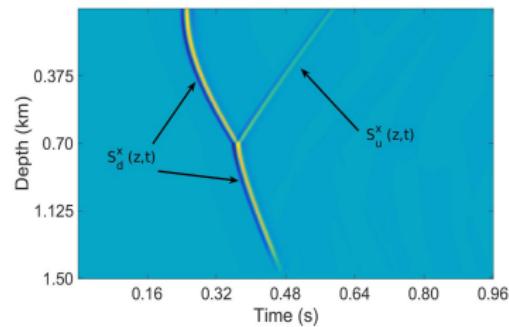
$S(x, z, t)$ wavefield



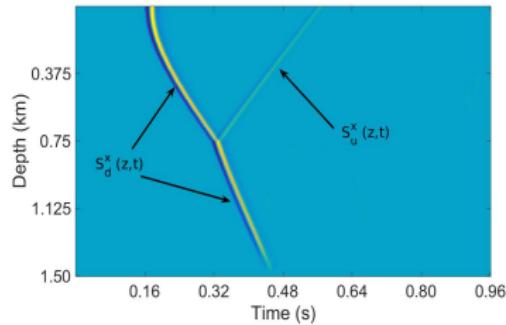
$S_x(z, t)$ at $x = 0.90 \text{ km}$



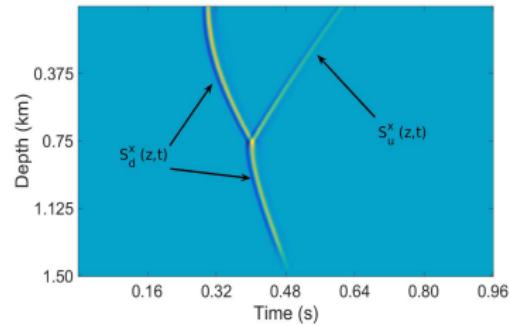
a) $x = 0.375$ km



b) $x = 0.90$ km



c) $x = 1.125$ km

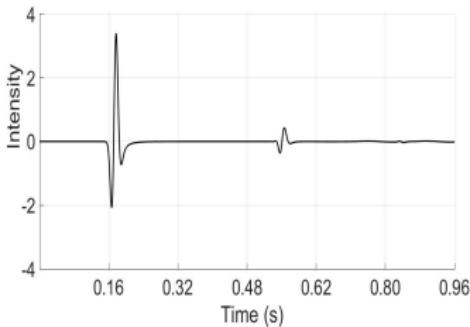


d) $x = 2.25$ km

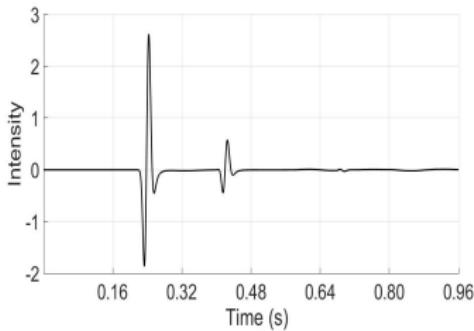
Source wavefield analysis

Algorithm for time-scale analysis of source wavefield

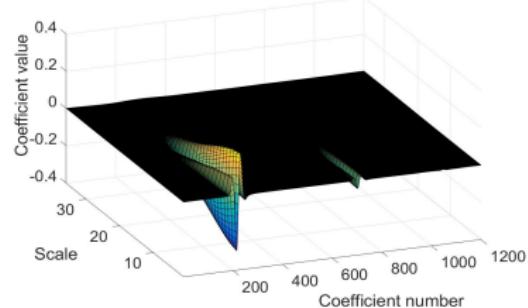
- ▶ From the source wavefield $S(x, z, t)$, select for each x the wavefield $S_x(z, t)$.
- ▶ Apply 1D CWT on $S_x(z, t)$ along t axis for each z ($S_{x,z}(t)$).



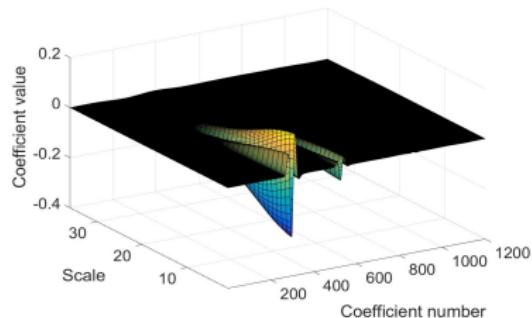
$S_{x=1.125,z}(t)$ at $z = 0$ km



$S_{x=1.125,z}(t)$ at $z = 0.45$ km



Coefficients of CWT

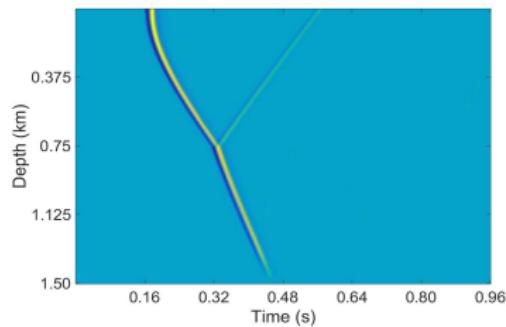


Coefficients of CWT

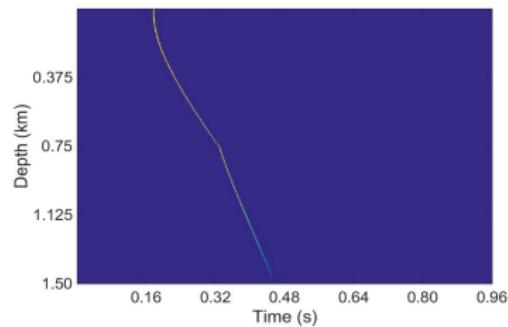
Source wavefield analysis

Algorithm for time-scale analysis of source wavefield

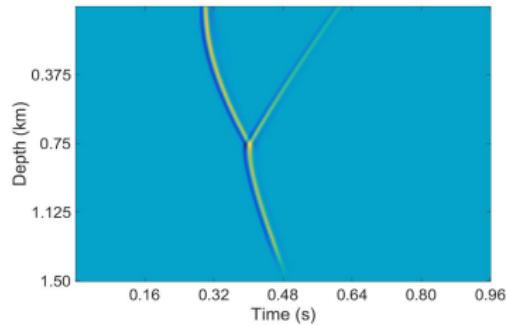
- ▶ From the source wavefield $S(x, z, t)$, select for each x the wavefield $S_x(z, t)$.
- ▶ Apply 1D CWT on $S_x(z, t)$ along t axis for each z ($S_{x,z}(t)$).
- ▶ Select the minimum value of the all coefficients and locate it in $S_{x,z}(t)$ and saved in a new wavefield $S_{x,z}^{new}(t)$. Two more points were taken before and after this point to improve the accuracy.



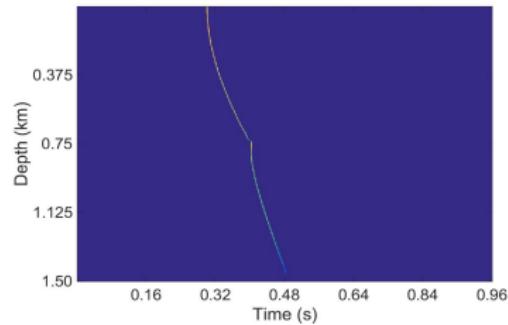
$S_x(z, t)$ at $x = 1.125$ km



$S_{x,z}^{new}(t)$ at $x = 1.125$ km



$S_x(z, t)$ at $x = 2.25$ km



$S_{x,z}^{new}(t)$ at $x = 2.25$ km

Source wavefield analysis

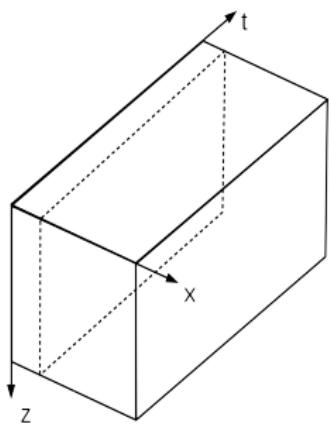
$S(x, z, t)$ wavefield video Separated $S(x, z, t)$ video

Receiver wavefield analysis

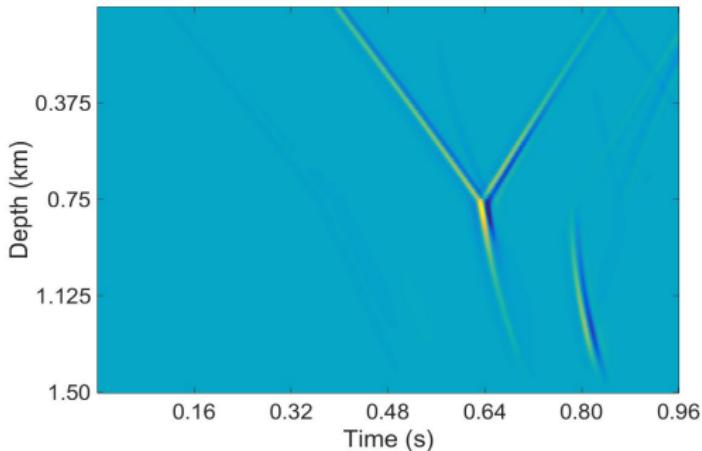
Algorithm for time-scale analysis of receiver wavefield

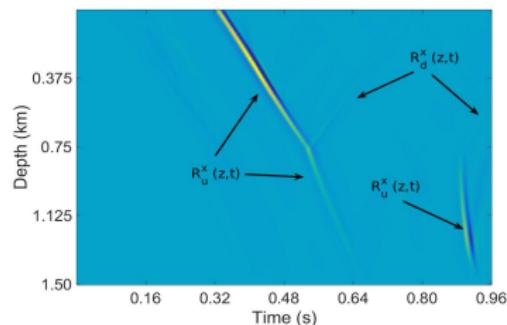
- ▶ From the receiver wavefield $R(x, z, t)$, select for each x the wavefield $R_x(z, t)$.

Receiver wavefield analysis

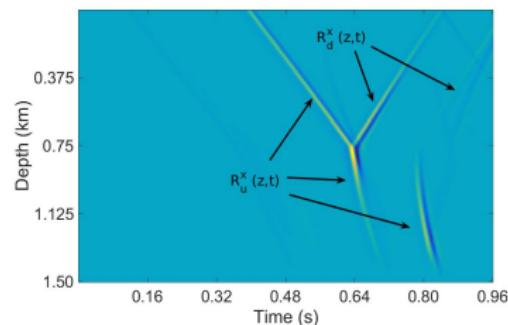


$R(x, z, t)$ wavefield

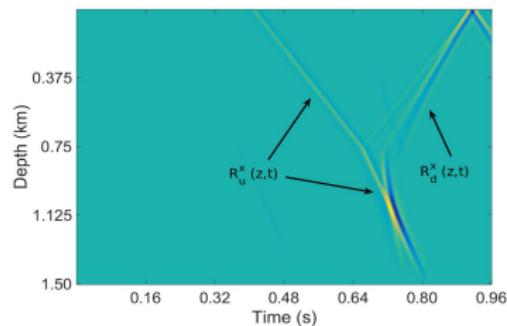




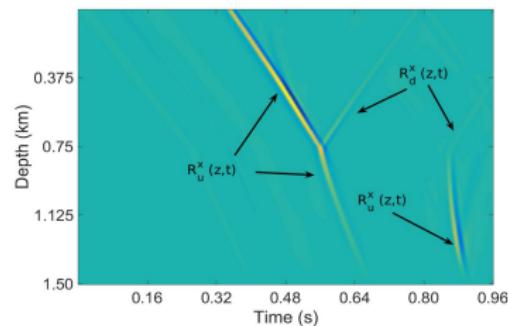
a) $x = 0.675 \text{ km}$



b) $x = 1.125 \text{ km}$



c) $x = 1.50 \text{ km}$



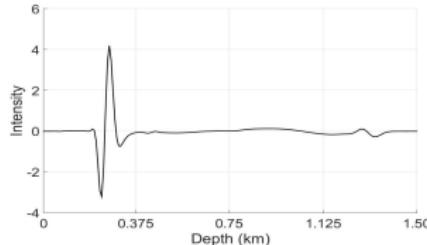
d) $x = 2.25 \text{ km}$

Receiver wavefield analysis

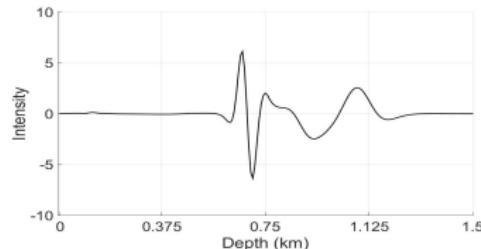
Algorithm for time-scale analysis of receiver wavefield

- ▶ From the receiver wavefield $R(x, z, t)$, select for each x the wavefield $R_x(z, t)$.
- ▶ Apply 1D CWT on $R_x(z, t)$ along z axis for each t ($R_{x,t}(z)$).

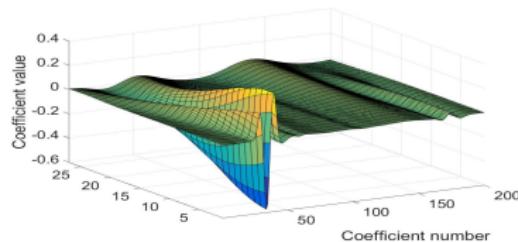
Receiver wavefield analysis



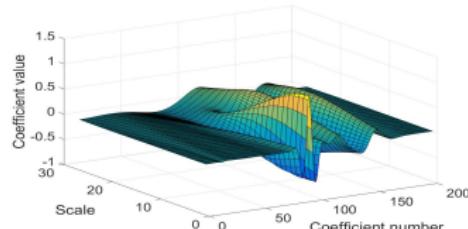
$R_{x=1.125,t}(z)$ at $t = 0.48$ s



$R_{x=1.125,t}(z)$ at $t = 0.66$ s



Coefficients of CWT

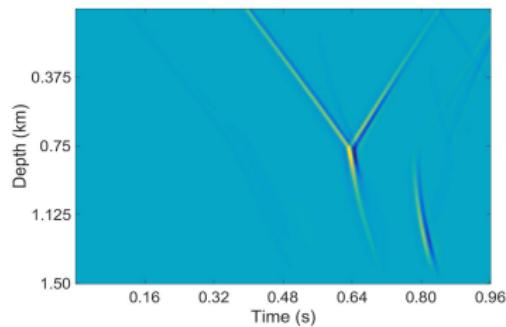


Coefficients of CWT

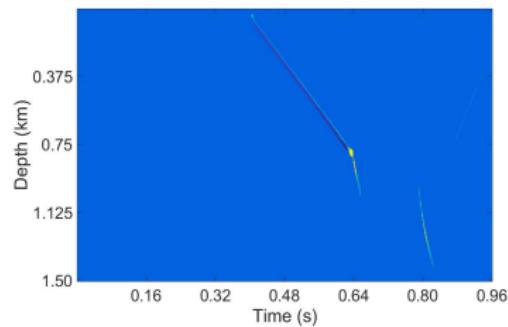
Receiver wavefield analysis

Algorithm for time-scale analysis of receiver wavefield

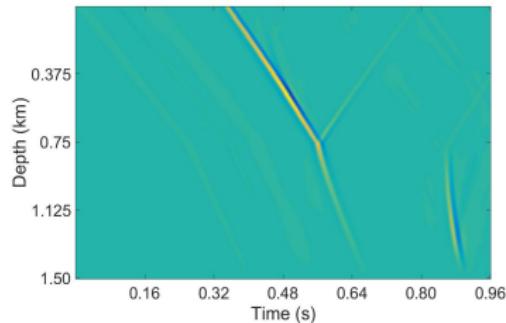
- ▶ From the receiver wavefield $R(x, z, t)$, select for each x the wavefield $R_x(z, t)$.
- ▶ Apply 1D CWT on $R_x(z, t)$ along z axis for each t ($R_{x,t}(z)$).
- ▶ Select the maximum absolute value of coefficients that corresponds to a coefficient with negative value and locate it in $R_{x,t}(z)$ and saved in a new wavefield $R_{x,t}^{new}(z)$. Two more points were taken before and after this point to improve the accuracy.



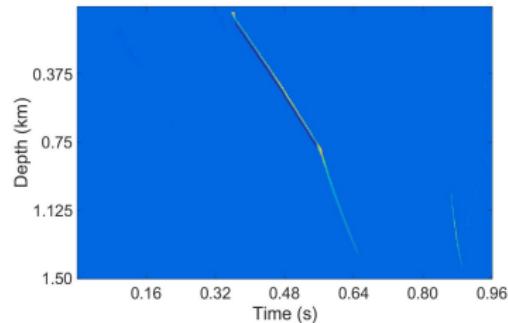
$R_x(z, t)$ at $x = 1.125$ km



$R_{x,t}^{new}(z)$ at $x = 1.125$ km



$R_x(z, t)$ at $x = 2.25$ km



$R_{x,t}^{new}(z)$ at $x = 2.25$ km

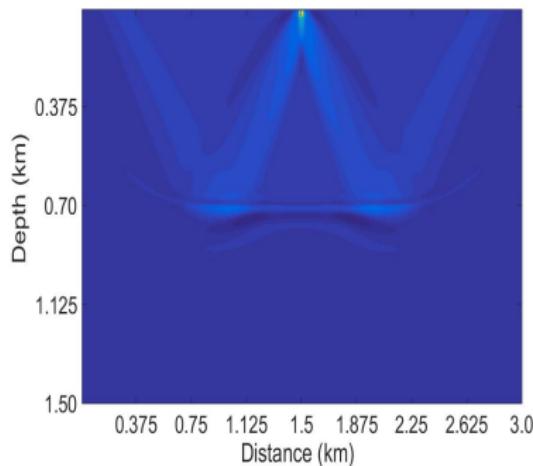
Receiver wavefield analysis

Receiver wavefield video Separated $R(x, z, t)$ video

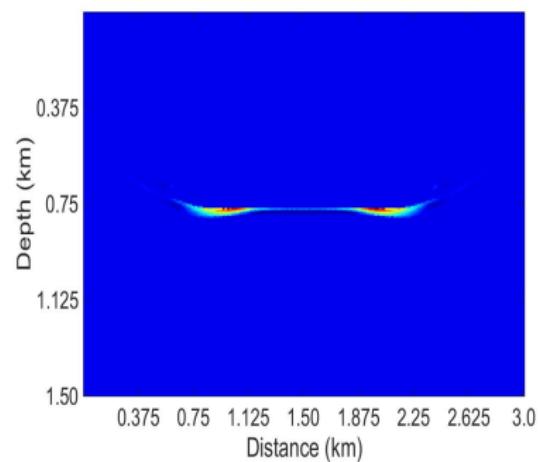
Separated source and receiver wavefields

Separated $S(x, z, t)$ video Separated $R(x, z, t)$ video

Cross-correlation image

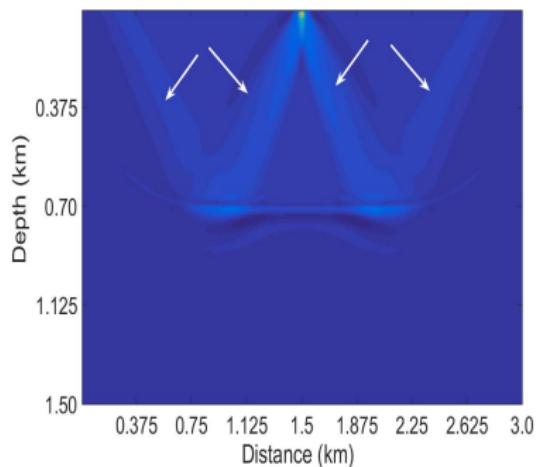


Conventional cross-correlation image

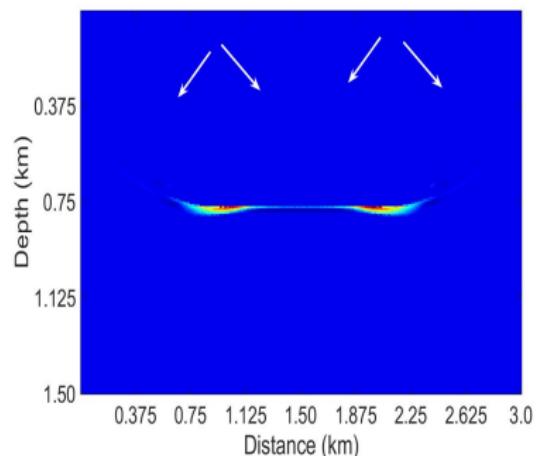


Wavefield separation cross-correlation image

Cross-correlation image

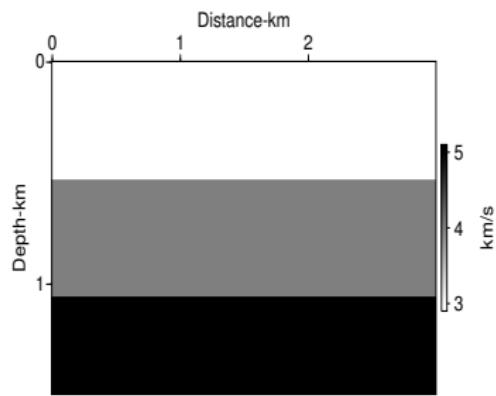


Conventional cross-correlation image

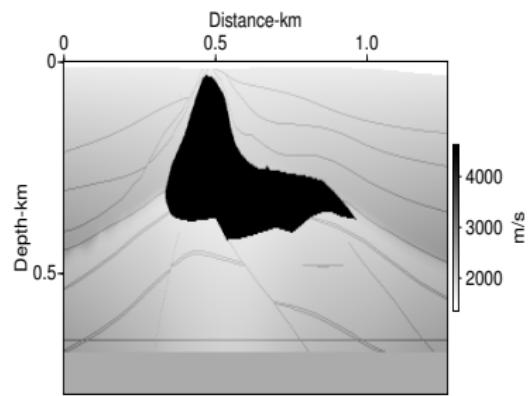


Wavefield separation cross-correlation image

Other synthetic models



Three-layer model



Small salt model

Three-layer model

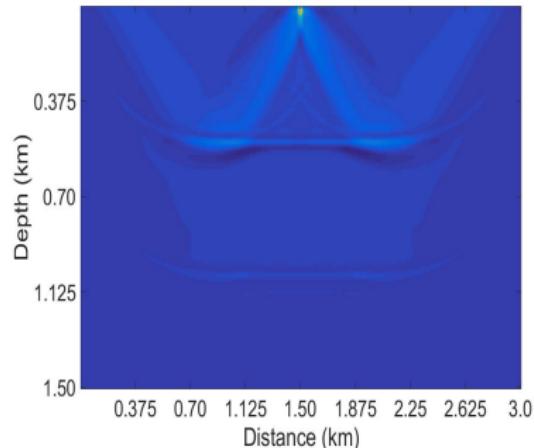
$S(x, z, t)$ video Separated $S(x, z, t)$ video

Three-layer model

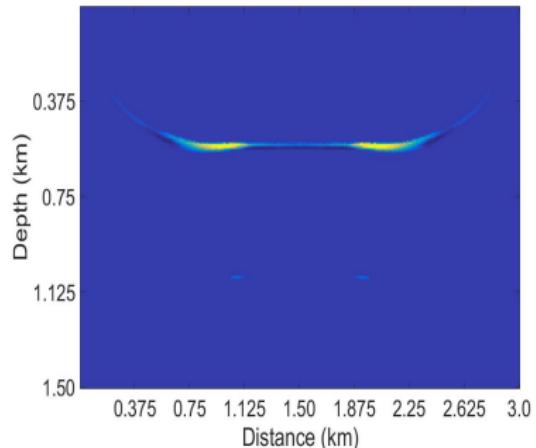
$R(x, z, t)$ video Separated $R(x, z, t)$ video

Three-layer model

Cross-correlation image



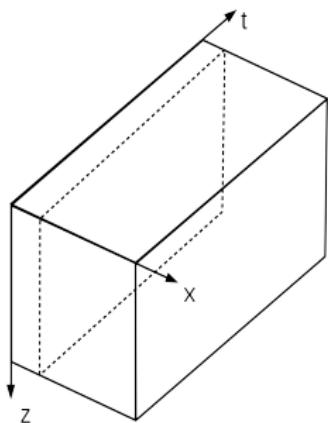
Conventional ZL-CC-IC



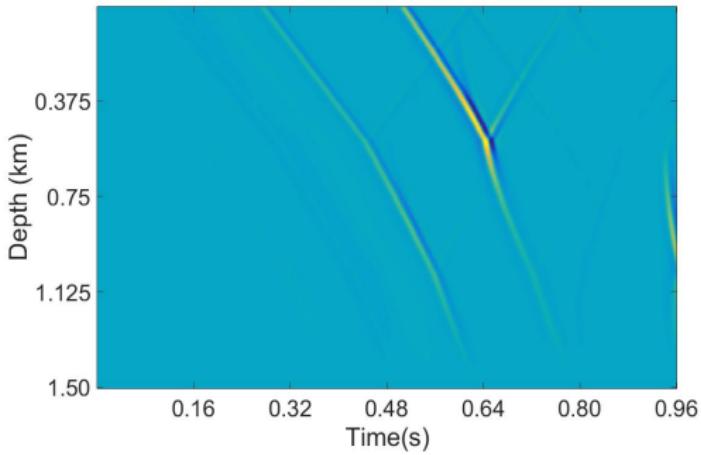
ZL-CC-IC with separated wavefield

Three-layer model

Receiver wavefield analysis



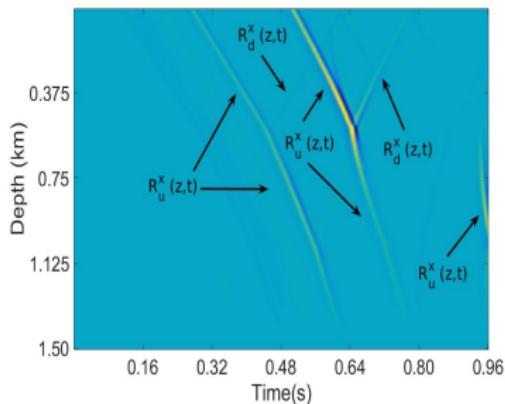
$$R(x, z, t)$$



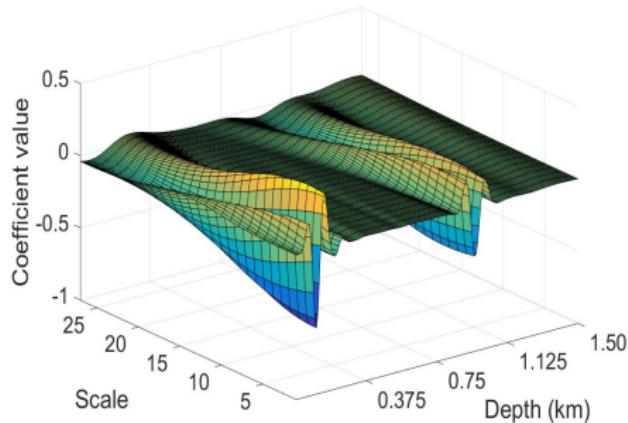
$$R(x, z, t) \text{ at } x = 0.9 \text{ km}$$

Three-layer model

Receiver wavefield analysis



$R(x, z, t)$ at $x = 0.9$ km



Coefficients CWT $R(x = 0.9, z, t)$ at $t = 0.54$ s

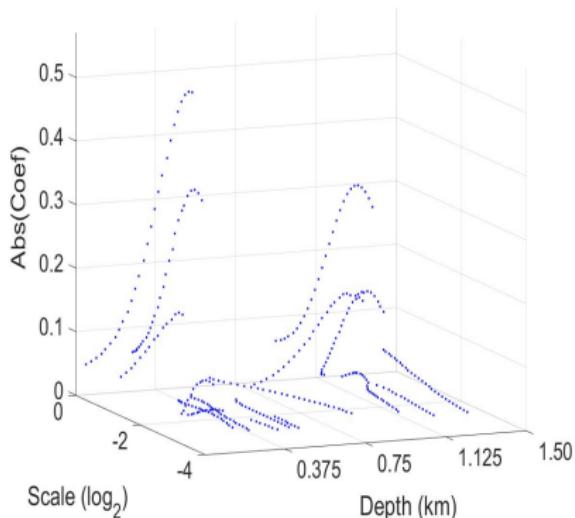
Wavelet transform modulus maxima (WTMM)

WTMM corresponds to the entire set of local maximum points of the absolute value of wavelet transform.

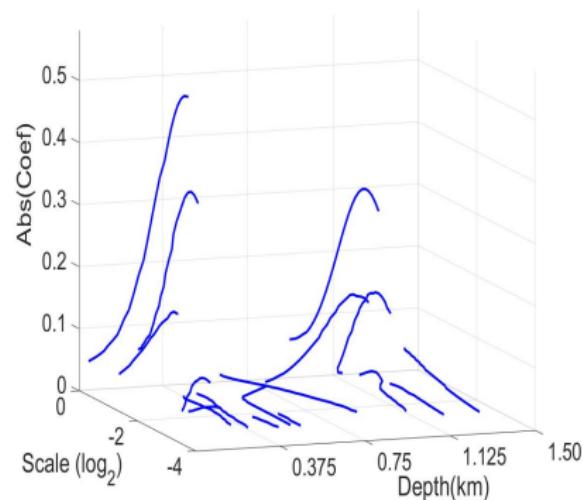
$$WTMM = \left\{ (u_0, s_0) \in (\mathbb{R}, \mathbb{R}^+), \frac{\partial |W_f(u, s)|}{\partial u} \Big|_{u=u_0, s=s_0} = 0 \right\}$$

The set of points of the WTMM concatenated through scales are known as maximum lines.

Wavelet transform modulus maxima (WTMM)



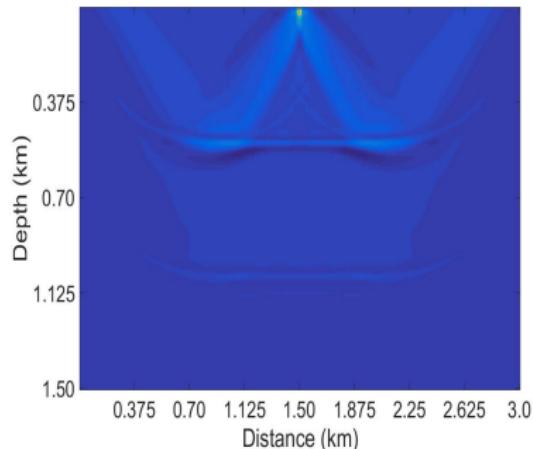
Local maximum points



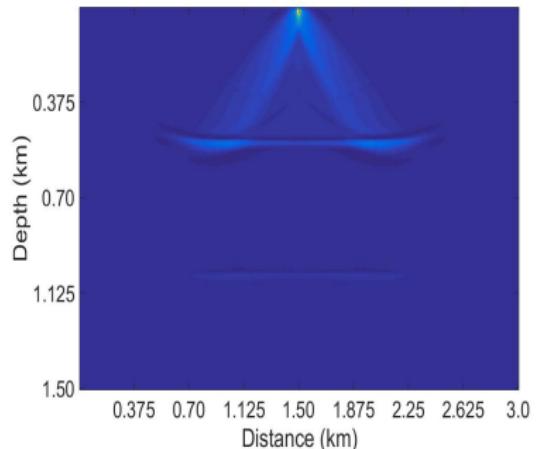
Maximum lines chaining

Three-layer model

Cross-correlation image



Conventional ZL-CC-IC



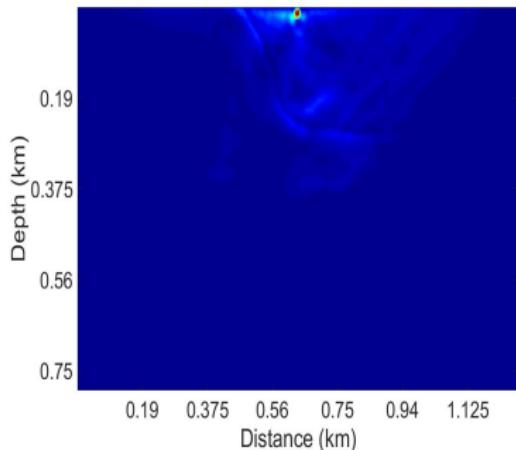
ZL-CC-IC with separated source wavefield

Small salt model

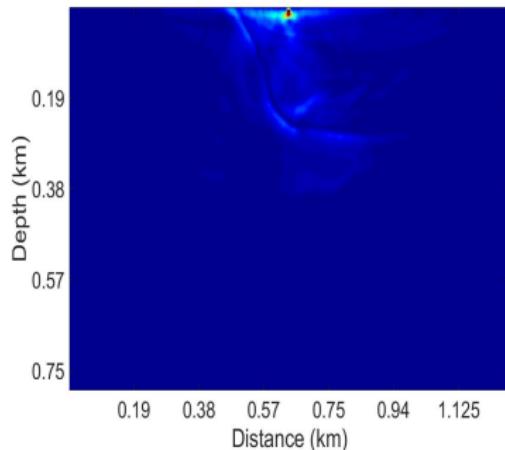
$S(x, z, t)$ video Separated $S(x, z, t)$ video

Small salt model

Cross-correlation image



Conventional ZL-CC-IC



ZL-CC-IC with separated source wavefield

Preliminary results

Paniagua, J.G. and Quintero, O.L., 2017, Attenuation of reverse time migration artifacts using Laguerre-Gauss filtering. Poster accepted in 79th EAGE Conference & Exhibition 2017, Paris.



Introduction

Reverse time migration (RTM) solves the two-way acoustic wave equation, by the propagation in time domain of the source wavefield in forward direction, and the receiver wavefield in backward direction. The RTM condition is obtained by applying the zero-lag cross-correlation (ZL-CC- δt) by examining the double summation of products of seismic amplitudes between the source and the receiver wavefields. One of them, summed in time domain and the other one, summed in the source domain. This imaging condition can be negatively affected by backscattered and turning waves in the modeling process. These artifacts are called migration artifacts. They are also called noise artifacts because they are the reflections points (Whitmore and Crowley, 2012). Correlation of direct waves, head waves and backscattered waves appear as low-frequency noise (Artifacts) which can hide important details in the image (Kachanov, 2006).

For small impedance contrasts the cross-correlation is a good approximation for the imaging condition.

However, for large impedance contrasts it is necessary to apply some modifications to obtain a better image (Kachanov, 2006; Tay et al., 2016).

Figure 1 shows the velocity model of 2D SEG-EAGE, its velocity gradient and the seismic image obtained by RTM and ZL-CC- δt . We can note that the artifacts are significantly reduced after applying velocity changes, which are evident near the flanks of the salt body (Jones and Davison, 2016).



Figure 1 2D SEG-EAGE model: a) Velocity model b) Velocity gradient c) Cross-correlation image

This artifacts can be eliminated or attenuated by modifications of the wave equation, using different imaging conditions or filtering techniques. Different techniques of post-processing of seismic images are used and the Laplacean filtering (LPF) is the most common (Kachanov and Vasyukov, 2001). This filtering technique has two major effects: i) it increases the high frequency noise and ii) it removes the low-frequency information (Cattaneo et al., 2007). However, the LPF is not able to remove the low-frequency noise in the seismic image, so we propose to use the LSOF to improve the zero-lag cross-correlation image. This Laguerre-Gauss spatial filter of complex values is a bandpass filter composed by a pure phase function and a Gaussian envelope in amplitude with a isotropic edge detection function. It is able to reduce the noise in the migration artifacts and the cross-correlation field obtained by applying the LSOF. This spatial filtering reduces the low-frequency noise and the subsurface structures.

In this paper, we show some features of the Laguerre-Gauss filtering in the post-processing of images obtained by cross-correlation imaging condition and its effect in the attenuation of low-frequency noise and the enhancement of the subsurface structures.

First, we present briefly the mathematical foundations of the Laguerre-Gauss spatial filtering (LSOF) and the LSOF properties. Then, we present the LSOF application to the seismic images to examine its effectiveness to reduce the low-frequency noise and enhance the subsurface structures. Then, we applied the LSOF on a complex synthetic model and present the results obtained by using the original and modified wave equations. Finally, we present the results obtained by applying the LSOF to the migration artifacts in simple and complex models with dipping reflectors.

Preliminary results

Paniagua, J.G. and Quintero, O.L., 2017, The use of Laguerre-Gauss transform in 2D reverse time migration imaging. Paper accepted in 15th International Congress of the Brazilian Geophysical Society and the EXPOGEf 2017, Rio de Janeiro.



The use of Laguerre-Gauss transform in 2D reverse time migration Imaging

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This paper was inspired by the interest shown by the "15th International Congress of the Brazilian Geophysical Society and the EXPOGEf 2017, Rio de Janeiro" on the use of the Laguerre-Gauss transform in seismic imaging. One of the main goals of this paper was to demonstrate the good performance of the Laguerre-Gauss transform in the reverse time migration (RTM) process. The second goal was to demonstrate the good performance of this technique and establish its physical meaning. The third goal was to demonstrate the good performance of the technique in the presence of large impedance contrasts. The fourth goal was to demonstrate the good performance of the technique in the presence of low frequency artifacts.

Abstract

Zero-Lag Cross-Correlation imaging condition (ZL-CCIC) is widely used in Reverse Time Migration (RTM) for the recovery of the subsurface images of the seismic. The presence of spatial low frequency noise, also called artifacts, is a common problem in seismic data. These artifacts are usually removed by applying filters that remove certain complex features. In this paper we propose a new post processing technique for the edge enhancement and reduction of low frequency artifacts. This paper demonstrates the good performance of this technique and establishes its physical meaning. We present the results obtained by applying the proposed technique to synthetic data with large impedance contrasts. The results obtained by applying the proposed technique are compared with the results obtained by applying the zero-lag cross-correlation imaging condition (ZL-CCIC), zero-lag cross-correlation imaging condition plus Laguerre-Gauss filtering (ZL-CCIG) and the cross-correlation imaging condition plus Laguerre-Gauss filtering (CCIG). The results obtained by applying the proposed technique are compared with the results obtained by applying the zero-lag cross-correlation imaging condition (ZL-CCIC), zero-lag cross-correlation imaging condition plus Laguerre-Gauss filtering (ZL-CCIG) and the cross-correlation imaging condition plus Laguerre-Gauss filtering (CCIG).

In the first part of the paper we show some results of the proposed technique in order to illustrate the importance of the proposed technique. In the second part of the paper we show some results obtained by applying the proposed technique to synthetic data with large impedance contrasts. The results obtained by applying the proposed technique are compared with the results obtained by applying the zero-lag cross-correlation imaging condition (ZL-CCIC).

Introduction

The imaging condition in reverse time migration (RTM) has been conventionally obtained by the zero lag cross-correlation condition, which is the product of the square of products of seismic amplitudes in the source and receiver locations. One of these conditions is the cross-correlation imaging condition. This condition is kinematically associated at the reflections due to the fact that it minimizes the cross-correlation between the wave space and time (Xu, et al., 2011), but the required employment of this condition in the reverse time migration produces kinematically correct images of the geometry of the subsurface structures (Christensen and Meltzer, 2006).

However, the image is contaminated with non spatial harmonics, such as the cross-correlation imaging condition of driving waves, head waves and bottom waves (Xu, et al., 2011).

For small impedance contrasts the cross correlation is a good approximation for the imaging condition. However,

for large impedance contrasts the low frequency artifacts increase (Xu, et al., 2011; Christensen and Meltzer, 2006; Tary, et al., 2014). In presence of strong velocity changes, strong impedance changes occur and the appearance of low frequency artifacts is greater (Tary, et al., 2014).

Christensen and Meltzer have proposed to eliminate or reduce the low frequency noise, which can be classified as a low frequency artifact, by applying a filter that is modified. (ii) Imaging condition approach: reflection wavefield is reconstructed by applying the imaging condition approach. (image is smooth) (Christen, et al., 2006).

In regard to the latter, the Laplacian filtering is the post-processing technique frequently used to attenuate the low frequency noise. This filtering is applied to the high-difference filtered seismic data, but it increases the high-frequency noise (Xu, et al., 2011; Christensen and Meltzer, 2006). Looking for a new technique that allows for the reduction of the low frequency noise without increasing the high-frequency noise, Paniagua and Sierra-Sosa (2016) proposed the use of the Laplace-Gauss transform in the post-processing stage. The image obtained by zero-lag cross-correlation imaging condition plus Laplace-Gauss filtering (ZL-CCIG) is better than the image obtained by zero-lag cross-correlation imaging condition plus Laplacian filtering. This image is called the Laplace-Gauss filter and it consists in a pure-phase function. In comparison, a Heaviside step function is a pure-phase gap when the amplitude in the horizontal angular direction and in the vertical direction is zero. Thus, the amplitude of the low frequency noise in the subsurface structures (surface patches) is reduced and the subsurface structures are more clearly defined. The Laplace-Gauss transform (LGT) is a good approximation to the Heaviside step function and the Laplace-Gauss filtering is a good approximation to the Laplacian filtering.

In a previous work we demonstrated that as post processing strategy, the Laguerre-Gauss Filtering (LG-CCIG) is a good approximation to the Laplacian filtering (LGT) and the high frequency noise is also removed, enhancing the quality of the seismic images. The use of the Laplace-Gauss transform allows for the quantification of the phenomena to obtain better results.

In this paper, we show some numerical test of the Laplace-Gauss filtering in the post processing of seismic images. We compare the results obtained by applying the cross-correlation imaging condition and the good performance of the Laplace-Gauss filtering and reduction of low frequency artifacts.

First, we compare and analyze the Fourier spectra obtained by applying ZL-CCIC and ZL-CCIG. The images obtained by the zero-lag cross-correlation (ZL-CCIC) and the zero-lag cross-correlation plus Laguerre-Gauss filtering (ZL-CCIG) are compared. Then, we show some results of the Laplace-Gauss filtering applied in synthetic datasets and finally, using some smoothed velocity

Fifteenth International Congress of the Brazilian Geophysical Society

Preliminary results

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Brasileira de Geofísica.

LAGUERRE-GAUSS FILTERS IN REVERSE TIME MIGRATION IMAGE RECONSTRUCTION

Juan Guillermo Paniagua, Daniel Sierra-Sosa and Olga Lucia Quintero

ABSTRACT

Reverse time migration (RTM) solves the acoustic or elastic wave equation by means of the extrapolation from source and receiver wavefields in time. A migrated image is obtained by applying a criteria known as imaging condition. The cross-correlation between source and receiver wavefields is the commonly used imaging condition. However, this imaging condition produces spatial low-frequency noise, called artifacts, due to the unwanted correlation of the diving head and backscattered waves. Several techniques have been proposed to reduce the artifacts occurrence. Derivative operators as Laplacian are the most frequently used. In this work, we propose a technique based on a spiral phase filter ranging from 0 to 2π , and a torsional amplitude bandpass filter, known as Laguerre-Gauss transform. Through numerical experiments we present the application of this particular filter on RTM images and its advantages over the standard imaging condition. We also present the effect of imaging condition by the zero-lag cross-correlation imaging condition, the Laplacian derivative and the Laguerre-Gauss filtering, showing their frequency features. We also present evidences not only with simulated noisy velocity fields but also by comparison with the model velocity field gradients that this method improves the RTM images by reducing the artifacts and notably enhance the reflective events.

Keywords: Laguerre-Gauss transform, zero-lag cross-correlation, seismic migration, Imaging condition

FILTROS DE LAGUERRE-GAUSS EM IMPRESSÃO DE IMAGEM DE MIGRAÇÃO DE TEMPO INVERSO

RESUMO

A migração reversa no tempo (RTM) resolve a equação da onda acústica ou elástica por meio da extrapolação a partir do campo de onda da fonte e do receptor no tempo. Uma imagem migrada é obtida aplicando um critério conhecido como condição de imagem. A correlação cruzada entre campos de onda da fonte e receptor é a condição de imagem comumente usada. No entanto, esta condição de imagem produz ruído espacial de baixa frequência, chamados artefatos, devido à correlação indesejada das ondas de meia-onda, cabeça e retroflexão. Várias técnicas têm sido propostas para reduzir a ocorrência de artefatos. Operadores derivados como Laplaciano são os mais utilizados. Neste trabalho, propomos uma técnica baseada em um filtro de fase espiral que varia de 0 a 2π , e um filtro passa-banda de amplitude torsional, conhecido como transformada de Laguerre-Gauss.através de experimentos numéricos, apresentamos a aplicação deste filtro particular em sis-

Preliminary results

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	Versión 2

Análisis de singularidad del campo receptor y su relación con la condición de imagen en RTM

Singularity analysis of receiver field and its relation to RTM Imaging condition

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Abstract: In this paper we introduce the singularity spectrum algorithm of a seismogram and analyze the features time-scale of the traces and the superposition of the scalograms of the complete set of traces. We also tested the post-processing of the images obtained by the ZLC (Zero-Lag Cross-Correlation) condition of artifacts that appears by Zero Lag Cross-Correlation Imaging condition (ZLCC) of the operator Reverse Time Migration (RTM). We also tested the post-processing of ZLCC by Laplacian Filter and Laplace-Gauss Filter. Finally, we evaluated the post-processing of images by correlation analysis and zero-lag cross-correlation. We also evaluated the post-processing of images by correlation analysis and zero-lag cross-correlation of the operator of Migration Tiempo Reverso. También evaluamos el post procesamiento de la imágenes obtenida vía condición de imágenes por correlación cero (Zero-Lag Cross-Correlation) y el filtro Laplace-Gauss (Paniagua and Sierra, 2016) comparando su capacidad de localización y remoción de artículos en términos de las características encontradas por análisis de singularidad.

Keywords: Wavelet Transform Modulus Maxima, Zero-Lag Cross-Correlation, Laplace-Gauss Filter, and Correlation.

Resumen: En este trabajo presentamos el algoritmo de círculo de singularidad de un seísmograma y analizamos las características tiempo-escala de las trazas y la superposición del escalamiento del conjunto completo de trazas, emergiendo las marcas principales en dominio del tiempo-escala que pueden aparecer por la condición de imágenes por correlación cero (Zero-Lag Cross-Correlation) o por correlación cero-lag (Zero-Lag Cross-Correlation) del operador de Migración Tiempo Reverso. También evaluamos el post procesamiento de la imágenes obtenida vía condición de imágenes por correlación cero (Zero-Lag Cross-Correlation) y el filtro Laplace-Gauss (Paniagua and Sierra, 2016) comparando su capacidad de localización y remoción de artículos en términos de las características encontradas por análisis de singularidad.

Palabras Claves: Transformada Wavelet Modulo Máximo, Zero-Lag Cross-Correlation, Filtro Laplace-Gauss y Migración inversa.

INTRODUCTION

Reverse time migration (RTM) is a very well-known technique for the retrieval of images of the subsurface from the solution of the acoustic wave equation for wavefield propagation through a

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Future work

- ▶ Analyze and study the extraction of information about the upgoing and downgoing components of source and receiver wavefields obtained through the CWT and WTMM.
- ▶ Analyze the features of the coefficients in CWT and WTMM of the signals in order to select and extract the information properly.
- ▶ Improve the algorithm to extract the relevant information about source and receiver wavefields in multi-layer synthetic models.
- ▶ Apply the proposed method in migrations with multiple shots.

Future work

- ▶ Extend the implementation of the algorithm to other complex synthetic models.
- ▶ Realize a singularity analysis of wavefields in order to find the relationship between the local maximum points and lines chaining, obtained by CWT and WTMM, with the Hölder exponent. (Our hypothesis is that there is a relationship between the local maximum points and the Hölder exponent)

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